

STRUCTURES AND TRANSFORMATIONS

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Automorphism of a structure preserves the structure

Conversely, if we start with the automorphism group, we can extract the invariants, which we can identify with the structure.

e.g. Klein's Erlangen Programme in Geometry

Geometry is the study of the invariants of some specified transformation group.

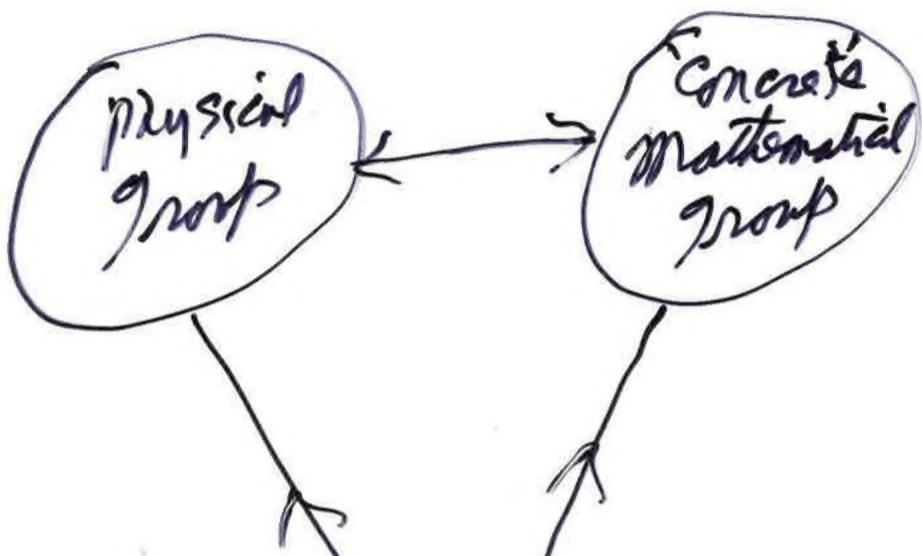
e.g. Euclidean $\text{geometry} \leftrightarrow \text{Euclidean group}$
Affine $\text{geometry} \rightarrow \text{Affine}$
Projective $\text{geometry} \leftrightarrow \text{Projective}$

More generally, morphisms between structures is the subject matter of Category theory.

Semantic Approach to Theories in Mathematical Physics

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Ex



Both satisfy the formal
actions of an abstract
group presented syntactically

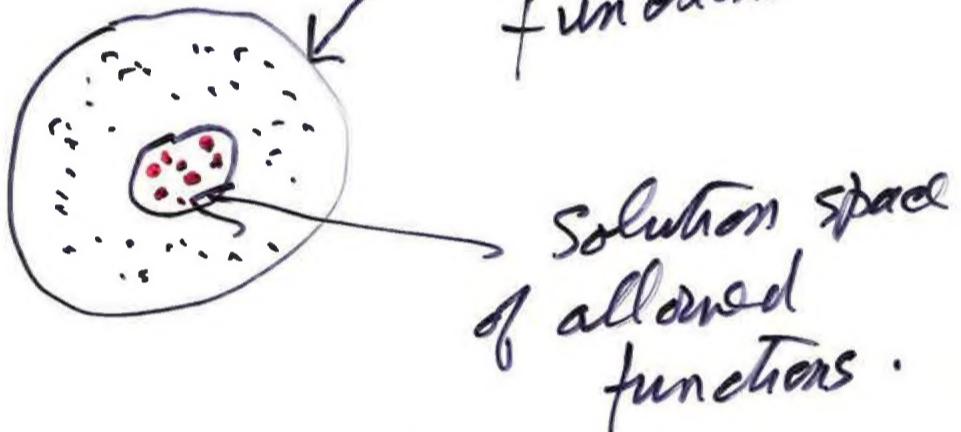
A THEORY OF THEORIES

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Possible states of a physical system associated with points in a function space.

Physical laws impose constraints on possible states (functions)

Ex Solutions of a mathematical equation



Symmetries of the physical theory are automorphisms of the solution space, i.e. map one solution onto another.

Limiting Relations

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Ex $Z = X + \alpha Y = 0 \quad (1)$

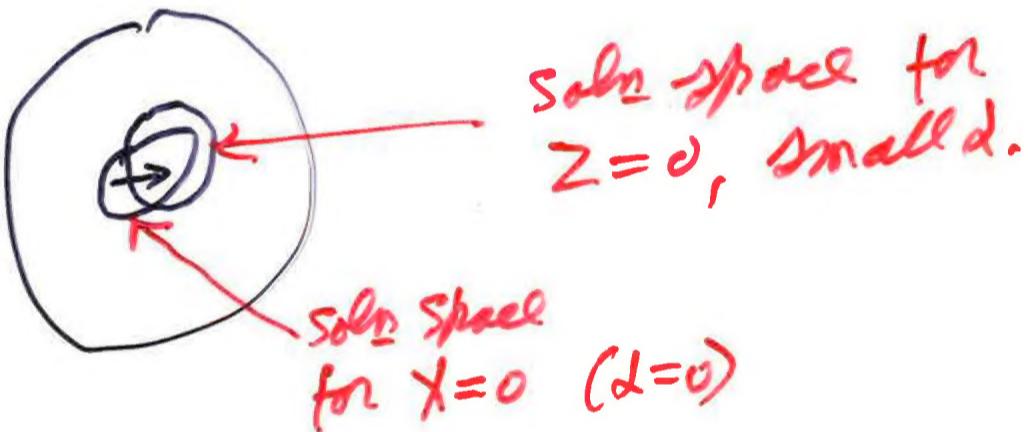
For $\alpha = 0$, eq. becomes.

$$X = 0 \quad (2)$$

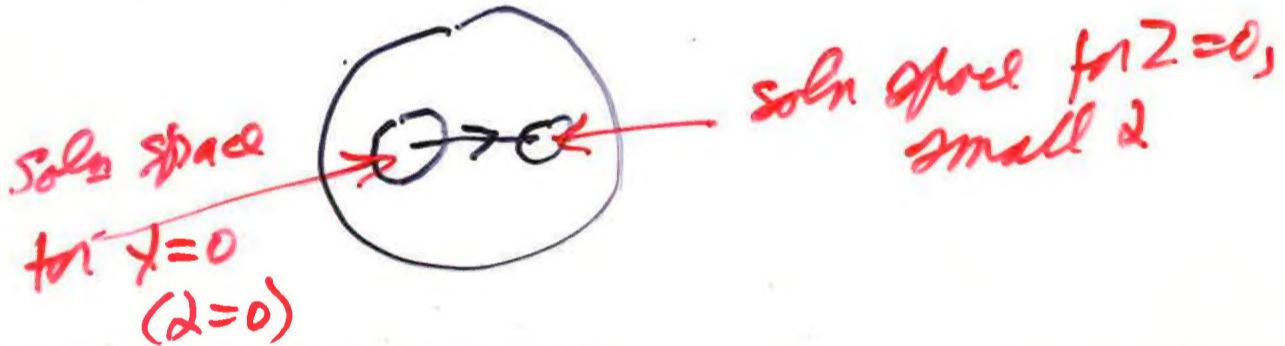
How are solutions of (1) for small α related to solutions of (2) where $\alpha = 0$.

Two possibilities:

(a) Continuous behaviour



(b) Discontinuous behaviour

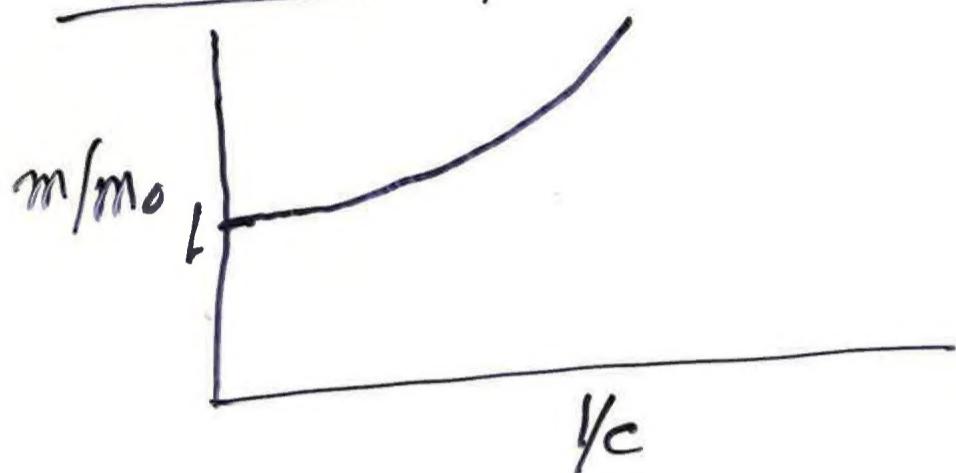


Special Relativity as $1/c \rightarrow 0$

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(a) Consider $m/m_0 = 1/\sqrt{1-v^2/c^2}$

For fixed v , this behaves in a continuous fashion as $1/c \rightarrow 0$



(b) Consider covariant metric tensor

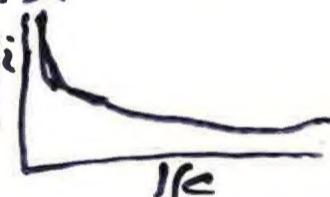
$$g_{\mu\nu} = \begin{pmatrix} -1/c^2 & & & \\ & -1/c^2 & & \\ & & -1/c^2 & \\ & & & -1/c^2 \end{pmatrix}$$

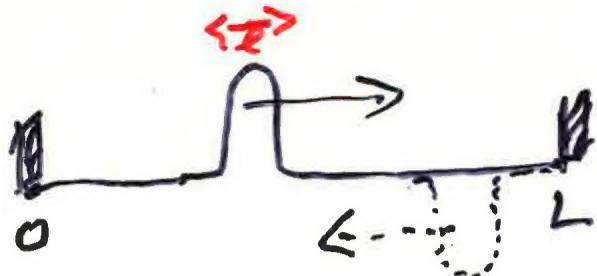
with $1/c = 0$ this becomes

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

which has no inverse, so contravariant metric tensor $g^{\mu\nu}$ does not exist.

g^{ii} exhibits discontinuity at $1/c = 0$





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Consider pulse of length l , unit amplitude travelling to the right

For resolution time T , mean displacement $\bar{\phi}$ satisfies

$$\bar{\phi} < \frac{l}{cT}, \quad \text{for } T > \frac{l}{c}$$

$$< 1$$

Indeed for fixed T , as $\frac{l}{c} \rightarrow 0$

$$\bar{\phi} \rightarrow 0$$

But for any value of $\frac{l}{c}$, however small, we can always choose T small enough ($< l/c$) so as to make $\bar{\phi} = 1$ for some value of x and t .

Compare fundamental mode with the zero sols at a particular point.

$$S_0^T (\sin \nu t - \phi) \text{ at } = \frac{1}{2} (1 - \cos \nu T) \rightarrow 0$$

as $\nu = \frac{c}{2L} \rightarrow \infty$.

So we have converged in the mean, but not pointwise convergence.

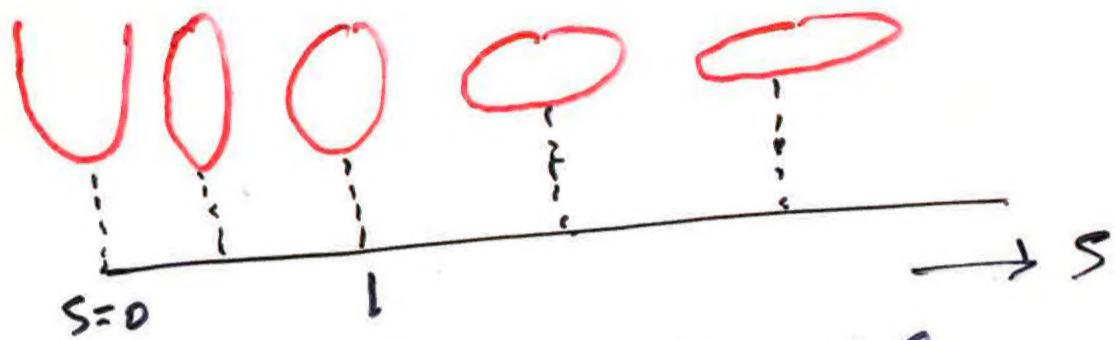
Conic Sections and Structural Stability

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Consider the following eq. in
2-dimensional Euclidean space

$$x^2 + sy^2 - 2x - 2y + 1 = 0$$

for variable parameter $s \geq 0$



For $s=0$ we have a parabola
for $s > 0$ ellipses of varying eccentricity
($s=1$ gives a circle).

What we are dealing with here is
a structural-valued function over
relative to the Euclidean group all
the figures are in different congruence
classes.

But relative to the Affine group
all the ellipses are equivalent, but not
the parabola, so we have structural
stability relative to the affine group
for all $s > 0$, with a singularity at $s=0$
(catastrophe)

HISTORY OF OPTICS

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QED (photons) h, c finite

↑
Maxwell's
eqns

$h=0, c$ finite

↑
Fresnel

same structure
- different ontology

↑
geometrical
optics

$\lambda=0, v=\infty, c$ finite

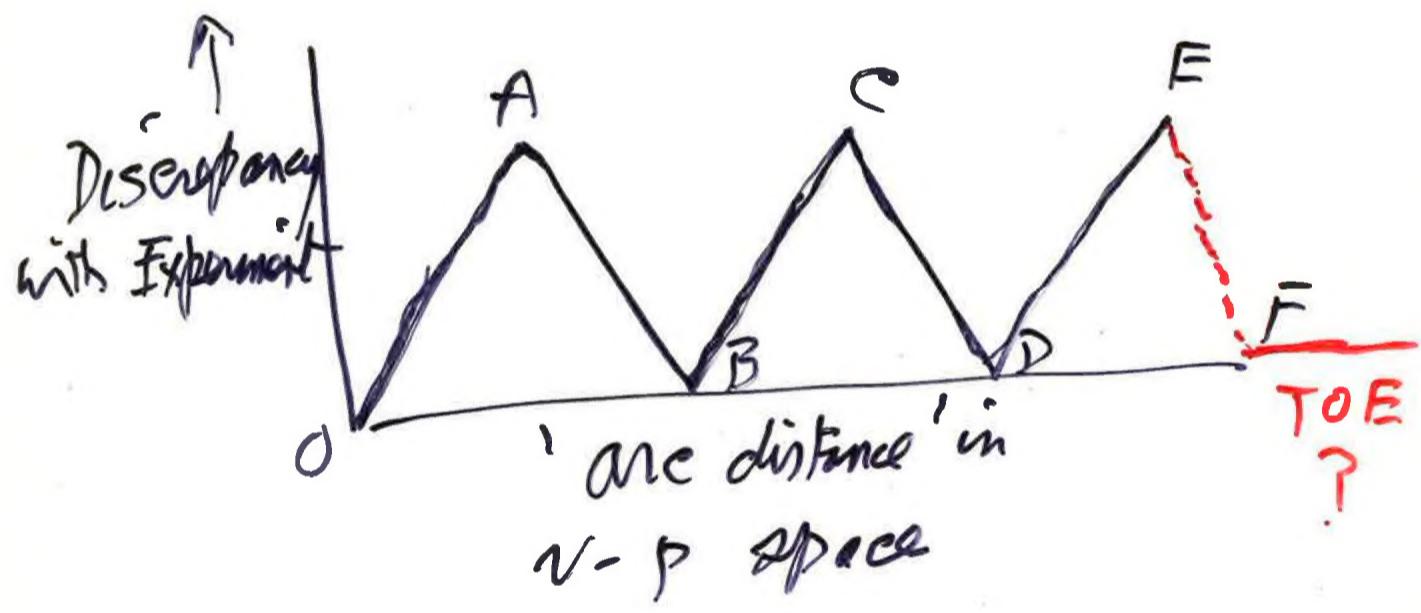
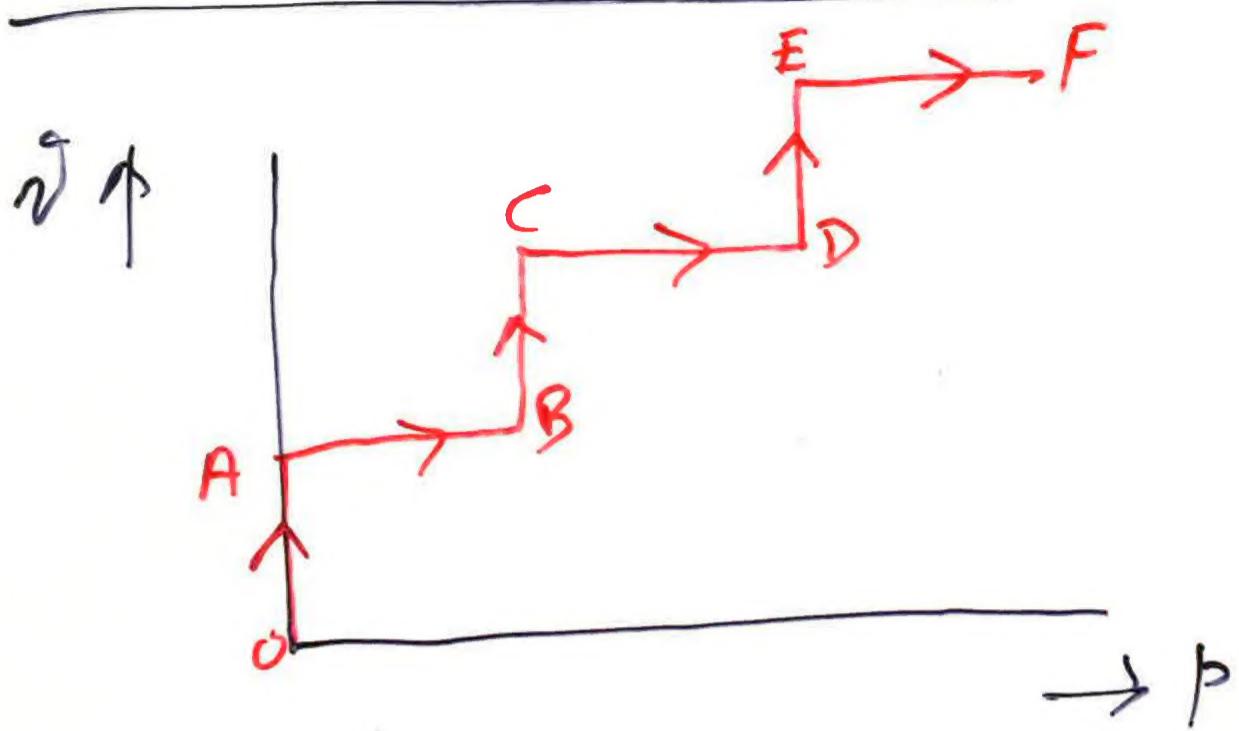
Ray optics
(line transformation)
Corpuscular theory
of light

Wave-front
optics
(Eikonal)
Wave theory
of light à la
Huygen's.

Hamilton
Gaussian optics
(point transformations,
spherical waves)

PROGRESS IN PHYSICS

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Gauge Transformations and Surplus Structure

$$\rho = e \psi^* \psi \text{ and } j = \frac{1}{2} i e \left(\psi^* \frac{d\psi}{dx} - \psi \frac{d\psi^*}{dx} \right)$$

are real quantities and represent physical magnitudes of charge and current density.

ρ and j are invariant under global phase transformations

$$\psi \rightarrow \psi e^{i\alpha}$$

But to retain local gauge invariance under $\psi \rightarrow \psi e^{i\alpha(x)}$ for the current density j , we must replace d/dx by $d/dx - i A(x)$ where A transforms as $A \rightarrow A + d/dx \alpha(x)$ and $j = \frac{1}{2} \cdot i e \left(\psi^* (d/dx - i A) \psi - \psi (d/dx + i A) \psi^* \right)$